Development of a Finite Difference Time Domain (FDTD) Model for Propagation of Transient Sounds in Very Shallow Water

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Abstract

This finite difference time domain (FDTD) model for sound propagation in very shallow water uses pressure and velocity grids with both three-dimensional Cartesian and two-dimensional cylindrical implementations. Parameters, including water and sediment properties, can vary in each dimension. Steady-state and transient signals from discrete and distributed sources, such as the surface of a vibrating pile, can be used. The cylindrical implementation uses less computation but requires axial symmetry. The Cartesian implementation allows asymmetry. FDTD calculations compare well with those of a split-step parabolic equation. Applications include modeling the propagation of individual fish sounds, fish aggregation sounds, and distributed sources.

1 Introduction

Underwater sounds include steady-state vessel noise, transient animal calls, and impulsive pile driving sounds. Many propagation models use frequency domain solutions with single frequencies. Any signal can be transformed into the frequency domain by representing it as a combination of infinite sine waves. Infinite sine waves do not represent transient and impulsive sounds efficiently because transient sounds contain many spectral components and impulsive sounds contain all frequencies requiring many intensive single-frequency computations. An alternate approach is to calculate sound propagation in the time domain. We developed a finite difference time domain (FDTD) model using sample grids of sound pressure and velocity in alternating time steps to model sound propagation in very shallow water (depth ≤ 10 m). Since all calculations are done in the time domain, this approach is particularly useful for modeling the propagation of transient and impulsive sounds. In this paper, we describe the model, compare propagation calculations for single-frequency sources to the calculations of the RAM program (Collins, 1995), and discuss sound propagation calculations for various transient sounds.

1.1 Finite Difference Time Domain Models

Yee (1966) developed the FDTD approach to model electromagnetic propagation. The approach was used by Botteldooren (1994) to model acoustic propagation in ducts, and by Sakamoto et al. (2002) to model acoustic propagation in indoor spaces. The FDTD approximates the differential equations governing propagation as finite difference equations. Spatial coordinates are computed on a grid (e.g., $x_1, x_2, x_3, ...$), and time t is taken in discrete steps (e.g., $t_1, t_2, t_3, ...$). Time derivatives are approximated as finite differences of time

$$\frac{\partial f(x,t)}{\partial t} \to \frac{\Delta f(x,t)}{\Delta t} = \frac{f(x,t_2) - f(x,t_1)}{t_2 - t_1}, \tag{1}$$

and spatial derivatives are approximated as finite differences of spatial coordinates

$$\frac{\partial f(x,t)}{\partial x} \to \frac{\Delta f(x,t)}{\Delta x} = \frac{f(x_2,t) - f(x_1,t)}{x_2 - x_1}. \tag{2}$$

The resulting finite difference propagation equations are solved for the time evolution of the acoustic parameters pressure and particle velocity, each of which depends on the spatial variations of the other parameter.

In an approach known as leapfrogging, spatial variations of pressure are used to calculate changes to the particle velocity, and spatial variations in particle velocity are used to calculate changes to the pressure. In the leapfrogging scheme, the particle velocity spatial grid points are halfway between the pressure grid points (see Figure 1). The pressure and particle velocity values are computed 1/2 time-step apart. The calculation alternates between particle velocity and pressure changes in each 1/2 time-step. We assume the seafloor to be an equivalent fluid and use its sound speed and density in the time-increment equations.

Figure 1 about here.

In order to compute propagation around barriers and in complicated indoor geometries, Sakamoto et al. (2002) used a pressure impulse as the source condition. They approximated an impulse with a pressure distribution that increases from zero to the maximum source pressure in 10 grid spaces from each direction. For sufficiently small grid spacing, this initial pressure behaves as an impulse.

1.2 Perfectly Matched Layer

One problem with finite difference calculations is the termination of the grid space. A wave reaching the end of the grid will be reflected when, in reality, waves continue to propagate into the distance. To prevent this numerical artifact a perfectly matched layer (PML) suppresses waves reflected from the end of the grid space (Teixeira and Chew, 1997). The PML is an artificial boundary with an attenuation that increases exponentially as the grid approaches a perfectly-reflecting termination. The gradual onset of the attenuation does not result in reflections as the wave enters the PML. By the time the wave propagates through the PML, reflects off the termination, and propagates back to the grid space, it attenuates sufficiently that it does not contribute to the total sound.

2 Theory

Our FDTD model begins with the linearized acoustic equations

$$\frac{1}{c^2}\frac{\partial p}{\partial t} + \rho_0 \nabla \cdot \vec{v} = 0, \tag{3}$$

and

$$\rho_0 \frac{\partial \vec{v}}{\partial t} + \nabla p = 0, \tag{4}$$

where c is the speed of sound, p the acoustic pressure, ρ_0 the ambient density, and $\vec{v}$ the particle velocity. We assume that there are no significant effects of ambient flow (currents). Equation (3) is the linearized equation of continuity, and Eq. (4) is the linearized momentum equation.

2.1 Cartesian Coordinates

Our three-dimensional FDTD implementation uses Cartesian coordinates x, y, and z, with z increasing in the downward direction. The grid has uniform spacing h. We assume the seafloor to be an equivalent fluid and use its sound speed and density in the time-increment equations. The water has a pressure-release surface and the grid terminates with PMLs in the other directions.

2.2 Cylindrical Coordinates

To reduce computation overhead, we assume axial symmetry and use cylindrical coordinates for range r and depth z. The two-dimensional grid has uniform spacing h. We assume a pressure-release surface and terminate the grids on the below and at high r with PMLs.

2.3 Impulse Propagation and Source Function

We use the FDTD impulse propagation method (Sakamoto et al., 2002) to propagate a pressure impulse from the source position(s) throughout the grid to the receiver positions. This propagated impulse response signal contains geometrical, reflective, and diffractive effects on the signal for all frequencies below the Nyquist frequency associated with the time step Δt. The propagated impulse response signal contains geometrical, reflective, and diffractive effects on the source signal. We then convolve the impulse response signal at the desired receiver position with the source signal function to obtain the propagated signal at the receiver position. This technique works for both steady-state and transient signals. For steady-state signals, the source function must have a duration long enough that the transient effects vanish before the signal ends at the receiver positions. Once the transient effects at the receiver position vanish, the signal at the receiver position has reached its steady state and can be used for the propagated steady-state signal. For transient source signals, the entire propagated signal is used at each receiver position. Multiple or distributed sources are represented with pressure impulses at each source point. The impulse response signals are convolved with the source functions to obtain propagated signals. Then propagated signals from each source point are combined with any necessary time differences added.

3 Comparison With RAM Calculations

To test our FDTD propagation calculations, we computed the propagation of steady-state constant-frequency signals for various frequencies and water depths and source/receiver geometries and compared them to calculations made using a split-step parabolic equation (Collins, 1993; 1994; Collins et al., 1996) calculation with the freely available Range-dependent Acoustic Model (RAM) program (Collins, 1995). We compared source signals with frequencies 250 Hz, 500 Hz, and 1000 Hz for several source and receiver positions and very shallow water depths and for very shallow sources and receivers in a semi-infinite ocean. Our FDTD model has good agreement with the RAM model for these frequencies and geometries. Figure 2 shows an example of our FDTD calculations compared to the RAM model for a water depth of 5 m and frequency 500 Hz.

Figure 2 about here.

4 Applications

We produced our FDTD model to predict the propagation of transient sounds in very shallow estuaries and rivers (depth ≤ 10 m) where we study fish sounds and the effects of anthropogenic noise on underwater animals. We (Sprague and Luczkovich, 2012a) used an earlier version of this model to calculate the propagation of transient *Cynoscion regalis* (weakfish) sounds in very shallow water with both level and sloped seafloors. We (Sprague and Luczkovich, 2012b) also used the FDTD model to calculate the propagation of weakfish sounds in order to estimate numbers of calling fish in aggregations. Another application of our model is to use the motion of a vibrating pile as the source function to calculate the propagation of the pressure and particle velocity produced during pile driving.

5 Discussion and Conclusions

We have developed an FDTD model for propagation calculations in very shallow water. We use an impulse propagation technique that can be adapted to a wide range of source functions and geometries. Our model produces propagation calculations for both steady-state and transient source functions, and it produces good agreement with the calculations produced using the RAM program (Collins, 1995).

There are some downsides to the FDTD approach, though. The calculations are computationally intensive, requiring long calculation times and large storage spaces for even small two-dimensional (cylindrical) geometries. Larger, asymmetrical geometries that require the three-dimensional implementation increase the calculation time and storage space requirements geometrically. Some of these limitations can be overcome using parallel computing platforms to produce simultaneous calculations of different parts of the geometry and large, dedicated data storage systems.

Time domain propagation models like the FDTD are particularly useful for transient signals that are compact in time but not in frequency. These transient signals include sounds of underwater animals such as fish that produce short-duration calls. We are interested in using propagation modeling to better understand the composition of calling fish aggregations, acoustic competition between fish species, and to explore possible relationships between the acoustic properties of various bathymetries and location of fish aggregations. Transient sounds produced by pile driving and other human activities are another useful application of this model. We look forward to applying our model to study these and other applications.

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Figure Legends

Figure 1. The grid used for our FDTD calculations. Pressure and velocity values are separated by a half grid-space to simplify finite difference calculations involving each variable. This figure shows the x-z plane, which has the same configuration as the y-z plane. In the cylindrical FDTD formulation, the r-z grid also has the same configuration.

Figure 2. A comparison of sound propagation calculations produced using the FDTD model to those produced using the split-step parabolic equation Range-dependent Acoustic Model (RAM; Collins, 1995). All calculations are for a 500 Hz constant frequency source at depth 2.38 m in a flat 5.00 m deep ocean. (A) Receiver at depth z_r = 0.998 m. (B) Receiver at depth z_r = 2.23 m. (C) Receiver at depth z_r = 2.84 m.

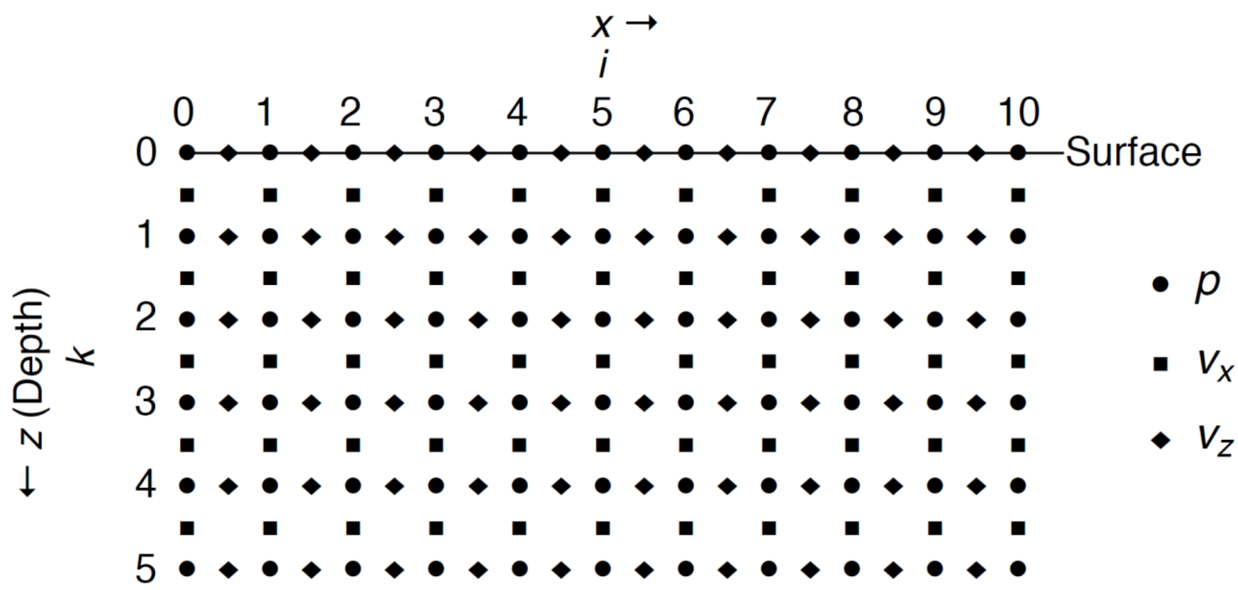

Figure 1

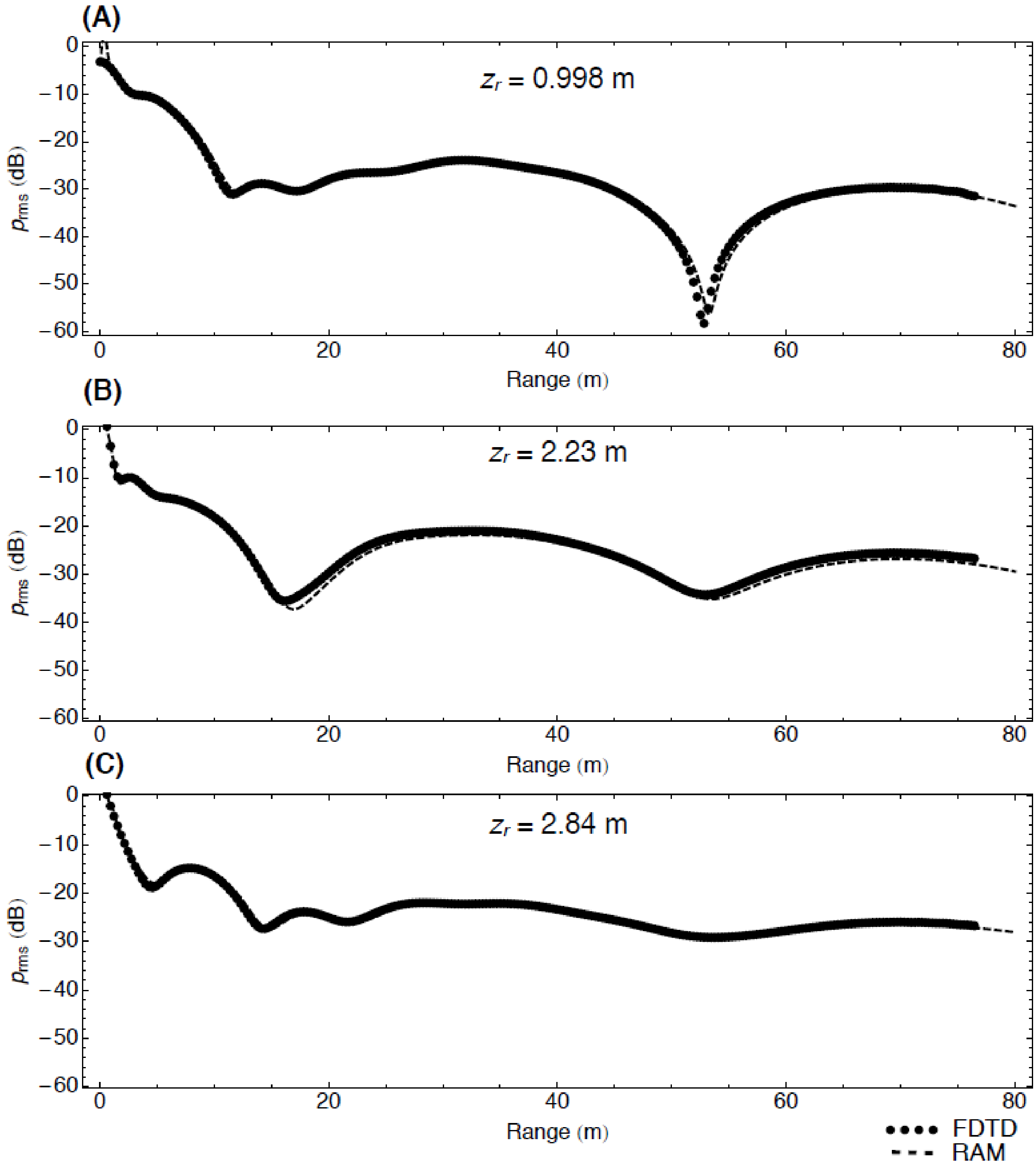
(A)
z_r = 0.998 m
p_{rms} (dB)
0
−10
−20
−30
−40
−50
−60
0
20
40
60
80
Range (m)
(B)
z_r = 2.23 m
p_{rms} (dB)
Range (m)
(C)
z_r = 2.84 m
p_{rms} (dB)
Range (m)
FDTD
RAM

Figure 2